Theory of magnetic field generation

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It is pointed out that fluctuating magnetic fields are generated on an ion time scale in laser-induced plasmas in the limit $\omega \ll \omega_{pe}$ where ω is the perturbation frequency and ω_{pe} is the electron plasma oscillation frequency. It is not the electron temperature perturbation that is crucial for the so-called magnetic electron drift vortex modes, as has been suggested previously. Some different electromagnetic and purely magnetic perturbations near the ion acoustic wave frequency in nonuniform plasmas have been proposed to be responsible for the generation of B fields. In the high-frequency range, propagation parallel to external gradients can give rise to instabilities of the radiation modes. [S1063-651X(96)05909-0]

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Despite a great deal of work in this direction, the phenomenon of magnetic field generation is still not well understood. It is generally believed that the main source of magnetic fluctuations in laser-induced plasmas is the first-order baroclinic effect, i.e., the term $\nabla n_0 \times \nabla T_{e1} \neq \mathbf{0}$ (where n_0 is the equilibrium plasma density and T_{e1} is the linear electron temperature perturbation) [1–9]. To the best of our knowledge, all the previous work on this topic discussed the possibility of magnetic field generation by electron motion alone. It has been assumed that the magnetic field and the associated instabilities can appear on an electronic time scale. The electron temperature perturbation is assumed to be adiabatic in collisionless plasmas. A lot of work on the linear and nonlinear propagation and instabilities of the so-called magnetic electron drift vortex (MEDV) modes has appeared in the literature [1–18] without considering ion response, with the assumption $\omega_{pi} \ll \omega \ll \omega_{pe}$ (where ω_{pi}, ω_{pe} are ion and electron plasma oscillation frequencies and ω is the perturbation frequency).

The displacement current is ignored and therefore Ampere's law implies $\nabla \cdot (n_0 \mathbf{V}_{e1}) = 0$ and hence $n_{e1} = 0$ (where $n_{e1}, \mathbf{V}_{e1}$ are the linear density and fluid velocity of electrons, respectively).

Since for hydrogen plasma $\omega_{pe}/\omega_{pi} \sim 43$, the assumption $\omega \ll \omega_{pe}$ suggests that ω can be near ω_{pi}. In this case, to ignore ion dynamics does not seem to be justified. Moreover for such low-frequency perturbations (where sometimes electron inertia is ignored) electrons can be considered isothermal instead of adiabatic. The stable surfacelike MEDV modes with density gradient along the x axis and wave vector $\mathbf{k} = \hat{\mathbf{y}} k$, widely discussed in the literature, has the linear dispersion relation

$$\omega^2 = \Gamma^2, \tag{1}$$

where $\Gamma^2 = (2/3a)\lambda^2 \kappa_n^2 v_{te}^2 k^2$, $\kappa_n = |d_x \ln n_0|$, $a = 1 + \lambda^2 k^2$, and $\lambda^2 = c^2/\omega_{pe}^2$.

Note that κ_n^{-1} is the density gradient scale length. Applying this result to some laser-induced plasmas, e.g., $n_0 \sim 10^{22}$ cm^{-3}, $T_{e0} \sim 1$ keV or $n_0 \sim 10^{20}$, $T_{e0} \sim 100$ eV, one notices that for short wavelengths $\kappa \sim 10^5$ in the presence of a steep density gradient $\kappa_n \sim 10^4$ cm, with local approximation $\kappa_n \ll k$, the frequency of perturbation ω turns out to be less than or near ω_{pi}. Hence one should take into account the ion response as well. The assumption $\omega_{pi} \ll \omega$ does not seem to be valid, in general.

Furthermore one notices that it is not the term $\nabla n_0 \times \nabla T_{e1}$ that is responsible for the generation of the magnetic field. The general argument is that, ignoring ion motion and the displacement current in Ampere's law, one obtains

$$n_0 \mathbf{V}_{e1} = -\frac{c}{4\pi e} \nabla \times \mathbf{B}_1 . \tag{2}$$

The curl of this equation actually gives

$$n_0 \nabla \times \mathbf{V}_{e1} + \nabla n_0 \times \mathbf{V}_{e1} = \frac{c}{4\pi e} \nabla^2 \mathbf{B}_1 . \tag{3}$$

If one divides Eq. (2) with n_0 and then takes the curl, the second term on the left-hand side (lhs) does not appear. The rhs remains the same because for surface waves $\nabla n_0 \cdot \nabla \times \mathbf{B}_1 = \mathbf{0}$.

Similarly the curl of the equation of motion,

$$m_e n_0 \partial_t \mathbf{V}_{e1} = -e n_0 \mathbf{E}_1 - \nabla p_{e1} , \tag{4}$$

does not give

$$\partial_t \nabla \times \mathbf{V}_{e1} = -\frac{e}{m_e} \nabla \times \mathbf{E}_1 + \frac{1}{m_e n_0} \nabla n_0 \times \nabla T_{e1} . \tag{5}$$

The thermoelectric term in Eq. (5) appears due to inappropriate ordering of mathematical steps, i.e., first dividing by n_0 and then taking the curl. Thus the term $(\nabla n_0 \times \nabla T_{e1})$ artificially becomes the source for the $\mathbf{B}_1$ field in the preceding studies. Here the subscripts naught (0) and one (1) denote the eqilibrium and perturbed quantities, respectively.

From our point of view, when we take the curl of Eq. (4), the term $(\nabla \times \nabla p_{e1})$ vanishes and hence the $\mathbf{B}$ field is not coupled with T_{e1}. Therefore, we reconsider the theory of magnetic field generation.

In this paper we present a mechanism of magnetic field generation in the frequency regimes $\omega^2 \ll \omega_{pe}^2$ and $\omega_{pe} < \omega$. For the sake of generality, perturbations are assumed to be in the xy plane. The ion temperature is ignored in both the frequency limits for $T_i \ll T_e$. First we consider the case $\omega \ll \omega_{pe}$. For these low-frequency fluctuations electrons are assumed to be isothermal.

The electron and ion fluid velocities are obtained from their equations of motion, respectively,

$$\mathbf{V}_{e1} = \frac{i}{\omega}\left\{ -\frac{e}{m_e}\mathbf{E}_1 - v_{te}^2 \frac{\nabla n_{e1}}{n_0} \right\} \tag{6}$$

and

$$\mathbf{V}_{i1} = \frac{ie}{m_i \omega}\mathbf{E}_1 . \tag{7}$$

The second term on the rhs of Eq. (6) comes from the term ∇p_{e1} in Eq. (4). We have assumed static equilibrium; therefore the Lorentz force does not enter in linear theory of unmagnetized plasmas. Substituting $\nabla\cdot(n_0 \mathbf{V_{e1}})$ from Eq. (4) into the electron continuity equation, one obtains

$$\frac{n_{e1}}{n_0} = -\frac{e}{m_e \alpha}(\kappa_n \hat{x}\cdot\mathbf{E}_1 + \nabla\cdot\mathbf{E}_1), \tag{8}$$

where $\alpha = \omega^2 - v_{te}^2 k^2$. Since T_{i0} is ignored, the ion continuity equation yields

$$\frac{n_{i1}}{n_0} = \frac{e}{m_i \omega^2}(\kappa_n \hat{x}\cdot\mathbf{E}_1 + \nabla\cdot\mathbf{E}_1). \tag{9}$$

Let us introduce in the usual manner the electrostatic and magnetic vector potential ϕ and $\mathbf{A}$ so that

$$\mathbf{E} = -\nabla\phi - \frac{1}{c}\partial_t \mathbf{A} \tag{10}$$

with Coulomb gauge $\nabla\cdot\mathbf{A} = 0$.

The quasineutrality gives rise to a coupling between A_{1x} and ϕ_1,

$$F(\omega)\left(q k^2 \phi_1 + \frac{i\omega}{c}\kappa_n A_{1x} \right) = 0, \tag{11}$$

where

$$F(\omega) = \frac{\omega^2 \omega_{pe}^2 + \alpha \omega_{pi}^2}{\omega^2 \alpha} \tag{12}$$

and $q = 1 - i(k_x \kappa_n / k^2)$.

For $\kappa_n = 0$, $F(\omega) = 0$ gives an electrostatic ion acoustic mode with dispersion relation $\omega^2 = c_s^2 k^2$, where $c_s^2 = T_e / m_i$. On the other hand, if $\kappa_n \neq 0$ and $\phi_1 = 0$, one may obtain a purely magnetic wave near ion acoustic frequency,

$$\omega^2 = \lambda_{De}^2 k^2 \omega_{pi}^2 = c_s^2 k^2, \tag{13}$$

where $\lambda_{De} = (T_0 / 4\pi n_0 e^2)^{1/2}$ is the electron Debye length. It is interesting to note that we obtain the same linear dispersion relation for $\phi_1 = 0$ or $A_{1x} = 0$ in nonuniform plasmas. The question arises whether the fluctuations near $\omega \sim c_s k$ in such plasmas represented by Eq. (11) are electrostatic or purely magnetic. Let $\phi_1' = e\phi_1 / T_{e0}$ and $A_{1x}' = (\omega / ck)(eA_{1x}/T_{c0})$. Then Eq. (11) implies

$$|A_{1x}'| \sim \left| \frac{k}{\kappa_n} \right| |\phi_1'|. \tag{14}$$

Since $k/\kappa_n \gg 1$ within the local approximation, we expect the shorter wavelength perturbations to behave dominantly as magnetic. Equation (11) suggests that one should find another relation between ϕ_1' and A_{1x}'. Moreover, $c_s k < \omega < v_{te} k$ is also possible. Therefore, for the sake of generality we retain the displacement current in Ampere's law,

$$\nabla\times\mathbf{B}_1 = \frac{4\pi}{c}\mathbf{J}_1 + \frac{1}{c}\partial_t \mathbf{E}_1 . \tag{15}$$

Now we look for the contribution of ion and electron vorticities to $\mathbf{B_1}$ field generation and let both ϕ_1, A_{1x} be nonvanishing. Then the Poisson equation gives instead of Eq. (11) the following relation:

$$[1 - qF(\omega)]\phi_1' = i\frac{\kappa_n}{k}F(\omega)A_{1x}', \tag{16}$$

where $A_{1y} = -k_x A_{1x}/k_y$ has been used. The curl of Eq. (15) gives

$$\begin{aligned}(c^2k^2 - \omega^2)\mathbf{B}_1 = 4\pi n_0 ec\{&(\nabla\times\mathbf{V}_{i1} + \kappa_n \hat{x}\times\mathbf{V}_{i1}) \\ &- (\nabla\times\mathbf{V}_{e1} + \kappa_n \hat{x}\times\mathbf{V}_{e1})\}.\end{aligned} \tag{17}$$

Using Eqs. (6) and (7) in Eq. (16), one obtains

$$\{-\omega^2 + c^2k^2 + q(\omega_{pe}^2 + \omega_{pi}^2)\}A_{1x}' = i\frac{\kappa_n k_y^2}{k^3}(\omega_{pe}^2 + \omega_{pi}^2)\phi_1' . \tag{18}$$

Equations (16) and (18) give the linear dispersion relation for these electromagnetic low-frequency waves,

$$\begin{aligned}&\{(\omega^2 - q\omega_{pi}^2)(\omega^2 - v_{te}^2 k^2) - q\omega_{pe}^2 \omega^2\}\{\omega^2 - c^2k^2 \\ &\quad - (\omega_{pe}^2 + \omega_{pi}^2)q\} \\ &= \left(\frac{\kappa_n k_y}{k^2}\right)^2 (\omega_{pe}^2 + \omega_{pi}^2)\{\omega^2(\omega_{pe}^2 + \omega_{pi}^2) \\ &\quad - \omega_{pi}^2 v_{te}^2 k^2\}.\end{aligned} \tag{19}$$

This is a sixth-order polynomial. In the limit ω^2, $\omega_{pi}^2 \ll \omega_{pe}^2$ it reduces to a quadratic equation,

$$\begin{aligned}&\left\{ (v_{te}^2 k^2 + q\omega_{pe}^2)(c^2k^2 + q\omega_{pe}^2) - \left(\frac{\kappa_n k_y}{k^2}\right)^2 \omega_{pe}^4 \right\}\omega^2 \\ &= \omega_{pi}^2 v_{te}^2 k^2 \left[(c^2k^2 + q\omega_{pe}^2)q - \left(\frac{\kappa_n k_y}{k^2}\right)^2 \omega_{pe}^2 \right].\end{aligned} \tag{20}$$

We choose plasma parameters of [9], i.e., $n_0 = 10^{22}$ cm^{-3}, $T_{e0} = 1$ keV, $\kappa_n \sim 3\times10^3$, and $k_x = k_y$. Then Fig. 1 shows that these electromagnetic perturbations can become unstable and the real frequency ω is of the order of Γ [3,5,7]. The growth rate of one of these modes γ_1 is of the order of a nanosecond ($\sim$laser pulse duration). Choosing a steeper density gradient $\kappa_n \sim 10^4$ cm^{-1}, in another example [8] with

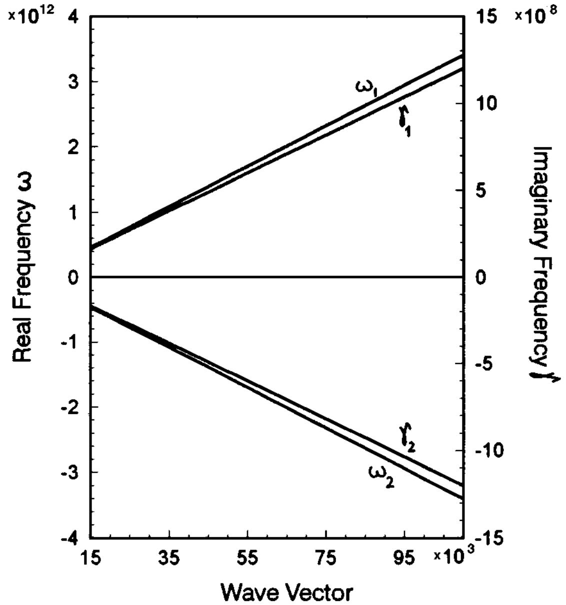

FIG. 1. Mode 1 is growing with wave vector and mode 2 is damped. Here $n_0 = 10^{22}$ and $T_{e0} = 1$ KeV.

$n_0 = 10^{20}$ cm^{-3} and $T_{e0} = 100$ eV, $k_x \sim (0.1) k_y$, one finds again one of the modes to be unstable with the same order of magnitude, i.e., the e-folding time of the instability is in a nanosecond range within the local approximation $k_x, k_y \gg \kappa_n$. The behavior of the real and imaginary frequencies is shown in Fig. 2 for this case.

On the other hand, $k_x = 0$ gives the stable surface wavelike modes

$$\omega^2 \sim \frac{\omega_{pi}^2 v_{te}^2 k^2 \left[(c^2k^2 + \omega_{pe}^2) - \left(\frac{\kappa_n k_y}{k^2} \right)^2 \omega_{pe}^2 \right]}{[v_{te}^2 k^2 c^2 k^2 + \omega_{pe}^2 (\omega_{pe}^2 + c^2 k^2)]}. \tag{21}$$

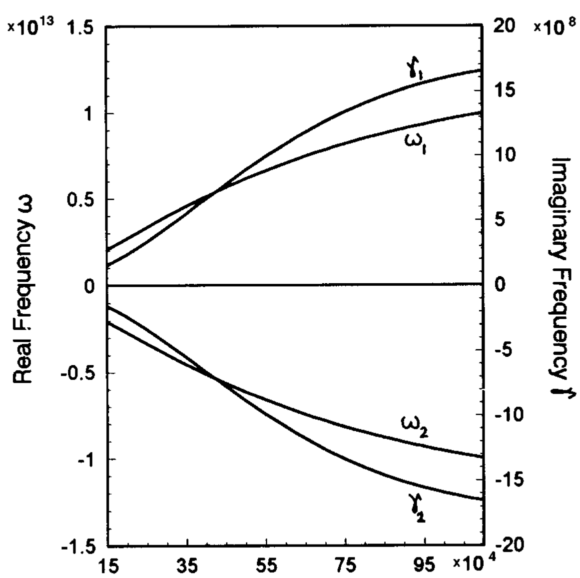

FIG. 2. Mode 1 is unstable and mode 2 is damped. Here $n = 10^{20}$ and $T_{e0} = 100$ eV.

Similarly, in the high-frequency range surface wavelike purely magnetic perturbations in an electron plasma were studied [19] assuming the term $(\nabla n_0 \times \nabla T_{e1})$ to be the source of $\mathbf{B}_1$. The adiabatic electron temperature fluctuations were obtained from the equation

$$\partial_t T_{e1} + \mathbf{V}_{e1} \cdot \nabla p_{e0} + (\gamma - 1) \nabla \cdot \mathbf{V}_{e1} = \mathbf{0}, \tag{22}$$

where γ is the ratio of specific heats and is equal to $\frac{5}{3}$ for ideal electron gas. The linear dispersion relation [Eq. (2) of [19]] can be written as

$$\omega^2 = \omega_{pe}^2 + c^2 k^2 + v_{te}^2 k^2 \frac{\kappa_n \kappa_T}{k^2} \frac{f(\omega, \kappa_n, \kappa_T)}{\left(1 - \frac{\omega}{\omega_{pe}} \right)^3}, \tag{23}$$

where v_{te} is the electron thermal velocity. The equation is not valid at resonance $\omega = \omega_{pe}$ where $k = 0$. This is the case of electrostatic plasma oscillations for the cold electron plasma. The local approximation requires $k \gg \kappa_n, \kappa_T$, and the validity of the classical physics demands $v_{te}^2 \ll c^2$. Hence the last term on the rhs in Eq. (23) is a very small contribution of temperature fluctuations to the ordinary mode radiation. But this small term can be responsible for the instability of these surface wavelike magnetic fluctuations.

From our point of view, the adiabatic electron temperature fluctuation can contribute to the electrostatic plasma waves but not to the radiation mode because $\nabla \times \nabla p_{e1} = \mathbf{0}$.

Electromagnetic radiation can propagate in the plasma with $k^2 > \omega_{pe}^2 / c^2$. In the limit $k^2 < \omega_{pe}^2 / c^2$, i.e., relatively longer wavelength, electrostatic plasma oscillations can propagate in hot electron uniform plasmas with the dispersion relation $\omega^2 \sim \omega_{pe}^2 + \frac{3}{2} v_{te}^2 k^2$. Since both $(\omega, \mathbf{k})$ regimes are different for these modes, they do not couple linearly in the plasma. However, the temperature fluctuations can couple with electrostatic plasma oscillations and cause electrostatic instabilities if certain conditions are satisfied.

In magnetized fusion plasmas, in general, $n_0 \sim 10^{15}$ cm^{-3} ($\omega_{pe} \sim 10^{14}$) and $T_{e0} \sim 10$ keV. Hence plasma waves and the ordinary (O) mode ($\omega^2 = c^2 k^2 + \omega_{pe}^2$) have clearly different regimes of wavelengths.

Here for clarity of physics, we only consider the purely magnetic perturbations in the high frequency regime which are more relevent to the present study. Substituting $(\nabla \times \mathbf{V}_{e1} + \nabla n_0 / n_0 \times \mathbf{A}_1)$ from Eq. (4) in Ampere's law, we obtain

$$\omega^2 = c^2 k^2 + \omega_{pe}^2 (1 - iP), \tag{24}$$

where $P = k_x \kappa_n / k^2$. This is the O-mode radiation in homogeneous plasma (i.e., for $P = 0$). We notice that the high frequency purely magnetic fluctuations can become unstable if they do not propagate as surface waves (i.e., if $k_x \neq 0$) with the condition $P < 0$ or $k_x \kappa_n < 0$.

For illustrative purposes, let us take $n_0 \sim 10^{20}$, $\kappa_n \sim 10^3$ and $k_x \sim 5 \times 10^4$. Then real and imaginary frequencies of dispersion relation (24) turn out to be, respectively, $\omega_r \sim 10^{15}$ sec^{-1} and $\gamma \sim 10^{12}$ sec^{-1}. The growth rate is ten times larger than predicted by Eq. (20) and a thousand times larger than estimates of Eq. (21). It is expected that the dissipative mechanisms can reduce the growth rate in such highly collisional plasmas.

In summary, we have reexamined the theory of magnetic field generation. In the frequency regime $\omega^2 \ll \omega_{pe}^2$, it has been shown that the role of ions has been overlooked in previous investigations. It is not the adiabatic electron temperature fluctuation which is the main source for B-field generation in nonuniform unmagnetized plasmas. Rather, it is the ion and electron density fluctuations and their vorticities which give rise to enormous magnetic field perturbations. Furthermore, a different purely magnetic mode near ion accoustic frequency has been pointed out. The coupling of ϕ_1 and A_{1x} can give rise to instabilities of these perturbations in nonuniform plasmas. The frequency range of Eq. (21) is the same as that of Eq. (1), whose derivation is not convincing to us. Therefore, we suggest that it is not the MEDV modes of Eq. (1) that are responsible for the generation of magnetic field; rather, these are due to electromagnetic modes of Eqs. (20) or (21).

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